Surface Area of Prisms and Cylinders

Section 9.2
Goal

- Find the surface areas of prisms and cylinders.
Key Vocabulary

- Prism
- Surface area
- Lateral face
- Lateral area
- Cylinder
A prism is a polyhedron with two congruent faces, called bases, that lie in parallel planes.

The other faces called lateral faces, are parallelograms formed by connecting the corresponding vertices of the bases.

The segments connecting these vertices are lateral edges.
Lateral Faces and Area

- The *lateral faces* of a prism are the faces of the prism that are not bases.
- The *lateral area* is the sum of the areas of the lateral faces.
Surface Area of a Prism

- To visualize the surface area of a prism, imagine unfolding it so that it lies flat. The flat representation of the faces is called a net.

- The surface area of a polyhedron is the sum of the areas of its faces. The surface area of a prism is equal to the area of its net.
Surface Area of a Prism

- **Method #1**

  - Calculate the areas of all the rectangles that form the faces of the prism.

  - Add the areas of all the faces to get the surface area.

  \[ S.A. = 2bh + 2bw + 2hw \]

<table>
<thead>
<tr>
<th>Congruent Faces</th>
<th>Dimensions</th>
<th>Area of Face</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Left face and right face</strong></td>
<td>8 in. by 5 in.</td>
<td>( 8 \times 5 = 40 \text{ in.}^2 )</td>
</tr>
<tr>
<td><strong>Front face and back face</strong></td>
<td>8 in. by 3 in.</td>
<td>( 8 \times 3 = 24 \text{ in.}^2 )</td>
</tr>
<tr>
<td><strong>Top face and bottom face</strong></td>
<td>3 in. by 5 in.</td>
<td>( 3 \times 5 = 15 \text{ in.}^2 )</td>
</tr>
</tbody>
</table>
Example Method #1

Area of Bases: \( 2A = bw \)

Lateral Areas: \( 2A = wh \) & \( A = bh \)

Add all the areas to get the surface area.

S.A. = \( 2bh + 2bw + 2hw \)

S.A. = \( 2(20) + 2(12) + 2(15) = 94 \text{ cm}^2 \)
Surface Area of a Prism

- **Method #2**

Surface area = \(2\text{(area of base)}\) + area of lateral faces

\[S.A. = 2B + Ph\]
Example Method #2

- S.A. = 2B + Ph

\[ B = \frac{1}{2} bh \quad P = 3 + 4 + 5 = 12 \ cm^2 \]
\[ B = 0.5(3)(4) \qquad h = 12 \ cm \]
\[ B = 6 \ cm^2 \quad Ph = 12 \cdot 12 = 144 \ cm^2 \]
\[ S.A. = 2(6) + 144 \]
\[ S.A. = 156 \ cm^2 \]
Surface Area of a Prism

**Words**  Surface area =
\[2(\text{area of base}) + (\text{perimeter of base})(\text{height})\]

**Symbols**  \[S = 2B + Ph\]
Example 1  Find Surface Area of a Prism

Find the surface area of the prism.

**SOLUTION**

1. Find the area of a triangular base.
   \[ B = \frac{1}{2} \cdot 4 \cdot 3 = 2 \cdot 3 = 6 \]

2. Find the perimeter of a base.
   \[ P = 3 + 4 + 5 = 12 \]

3. Find the height of the prism. In the diagram, \( h = 2 \).
Example 1  Find Surface Area of a Prism

4. Use the formula for surface area of a prism.

\[ S = 2B + Ph \]  \hspace{1cm} \text{Formula for the surface area of a prism}

\[ = 2 \cdot 6 + 12 \cdot 2 \]  \hspace{1cm} \text{Substitute 6 for } B, 12 \text{ for } P, \text{ and } 2 \text{ for } h.

\[ = 12 + 24 \]  \hspace{1cm} \text{Multiply.}

\[ = 36 \]  \hspace{1cm} \text{Add.}

\textbf{ANSWER}  \hspace{2cm} The surface area of the prism is 36 square meters.
Your Turn: Find Surface Area of Prisms

Find the surface area of the prism.

1. \[ \text{ANSWER} \quad 72 \text{ in.}^2 \]

2. \[ \text{ANSWER} \quad 236 \text{ ft}^2 \]

3. \[ \text{ANSWER} \quad 144 \text{ cm}^2 \]
Cylinder

- A **cylinder** is a solid with congruent circular bases that lie in parallel planes.
- The **altitude**, or **height** of a cylinder is the perpendicular distance between its bases.
- The **radius** of the base is also called the radius of the cylinder.
Surface area of cylinders

- The **lateral area of a cylinder** is the area of its curved surface.
- The lateral area is equal to the product of the circumference and the height, which is $2\pi rh$.
- The entire **surface area of a cylinder** is equal to the sum of the lateral area and the areas of the two bases.
Net of a Cylinder

Lateral Area is just a Rectangle!

Lateral Area = \(2\pi rh\)

Base Area is area of a circle

\[ B = \pi r^2 \]

Circumference of the circle
Surface Area of a Cylinder

\[ \text{Surface area} = 2\pi r h + 2\pi r^2 \]

\[ \text{S.A.} = 2\pi rh + 2\pi r^2 \]

or

\[ \text{S.A.} = 2\pi rh + 2B \]
Example Surface Area of a Cylinder

Lateral Area = $2\pi rh$
= $2\pi(6)(9)$
= $108\pi \text{ft}^2$
= $339.3 \text{ft}^2$

Area of Base
$B = \pi r^2$
= $\pi(6)^2$
= $36\pi \text{ft}^2$

$2B = 2(36\pi) \text{ft}^2$
= $226.2 \text{ ft}^2$

$SA = \text{Lateral Area} + 2B$
= $339.3 \text{ft}^2 + 226.2 \text{ft}^2$
= $565.5 \text{ft}^2$
Surface Area of a Cylinder

**Words**
Surface area = 2(area of base) + (circumference of base)(height)

**Symbols**
\[ S = 2B + Ch = 2\pi r^2 + 2\pi rh \]
Example 2  Find Surface Area of a Cylinder

Find the surface area of the cylinder. Round your answer to the nearest whole number.

SOLUTION

The radius of the base is 3 feet and the height 4 feet. Use these values in the formula for surface area of a cylinder.

\[ S = 2\pi r^2 + 2\pi rh \]

Write the formula for surface area.

\[ = 2\pi(3^2) + 2\pi(3)(4) \]

Substitute 3 for \( r \), and 4 for \( h \).

\[ = 18\pi + 24\pi \]

Simplify.

\[ = 42\pi \]

Add.

\[ \approx 132 \]

Multiply.

ANSWER  The surface area is about 132 square feet.
Example 3  Find Lateral Area

About how much plastic is used to make a straw that has a diameter of 5 millimeters and a height of 195 millimeters?

**SOLUTION**

The straw is a cylinder with no bases. Use the formula for the surface area of a cylinder, but do not include the areas of the bases.

The diameter is 5 millimeters. So the radius is \( \frac{5}{2} = 2.5 \).

\[
Lateral\ area = 2\pi r h \\
= 2\pi (2.5)(195) \quad \text{Surface area formula without bases.} \\
= 975\pi \quad \text{Substitute 2.5 for } r, \text{ and 195 for } h. \\
\approx 3063 \quad \text{Simplify.} \\
\]

\( \approx 3063 \) \quad \text{Multiply.}
Example 3  Find Lateral Area

ANSWER  The straw is made with about 3063 square millimeters of plastic.
Your Turn:

Find the area described. Round your answer to the nearest whole number.

1. surface area

![Cylinder diagram](image1)

ANSWER 151 in.²

2. surface area

![Cylinder diagram](image2)

ANSWER 603 ft²

3. lateral area

![Cylinder diagram](image3)

ANSWER 13 m²
Example 4

Find the surface area of each figure

a. \( S = 2\pi r^2 + 2\pi rh \)
   \[
   = 2\pi(4^2) + 2\pi(4)(6)
   = 80\pi \text{ in}^2 \approx 251.2 \text{ in}^2
   \]

b. \( S = 2B + Ph \)
   \[
   = 2\left(\frac{1}{2} \cdot 8 \cdot 3\right) + (18)(10)
   = 204 \text{ ft}^2
   \]
Example 5

Find the surface area of each figure

a. \( S = 2\pi r^2 + 2\pi rh \)
   
   \[
   \begin{align*}
   S &= 2\pi (15^2) + 2\pi (15)(3) \\
   &= 540\pi \text{ in}^2 \approx 1695.6 \text{ cm}^2
   \end{align*}
   \]

b. \( S = 2B + Ph \)
   
   \[
   \begin{align*}
   S &= 2\left(\frac{1}{2} \cdot 7 \cdot 6\right) + (21)(10) \\
   &= 252 \text{ cm}^2
   \end{align*}
   \]
3. All outer surfaces of a box are covered with gold foil, except the bottom. The box measures 6 in. long, 4 in. wide, and 3 in. high. How much gold foil was used? 84 in$^2$
Assignment

- Pg. 487: #8-58 odd